Postseismic signature of the 2004 Sumatra earthquake on low-degree gravity harmonics

V. Cannelli, D. Melini, A. Piersanti, and E. Boschi

1. Introduction

[1] We perform an extensive analysis of the low-degree gravity field harmonics measured by the GRACE mission, in order to find a signature of the postseismic relaxation following the 2004 Sumatra earthquake. We find a statistically significant perturbation in the secular trend of low-degree zonal coefficients \( J_{l} \) in correspondence of the 2004 Sumatra earthquake and a similar perturbation, but with weak associated statistical significance, also in the nonzonal coefficients. Technical features and results of such analysis are discussed. The time-dependent postseismic evolution of harmonic coefficients is modeled for various asthenosphere viscosity values, using a theoretical model of global postseismic deformation. The observed change in secular trend is found to be consistent with our modeling results but it cannot be used to discriminate between viscosities. A forward modeling of the perturbations to time-dependent zonal variation rates following the Sumatra earthquake for various asthenosphere viscosities is provided. As a result, an evident signature of the Sumatra earthquake on \( J_{l} \) time series is expected for asthenospheric viscosity values below \( 10^{18} \) Pa s. Therefore, long term \( J_{l} \) time histories from satellite laser ranging will be able to put constraints on the asthenosphere viscosity, if such a signature is evidenced from data or, at least, put lower limits if no significant perturbation will be observed.

harmonic coefficients temporal evolution as a function of asthenosphere viscosity. In fact, at least in principle, it should be possible to perform an indirect estimate of asthenosphere viscosity from geodetic data, if a significant deviation of the time series from its secular trend occurred, as speculated by Cannelli et al. [2007] for the degree 2 zonal coefficient. GRACE observations, from this point of view, represent a good candidate because of the length and continuity of the provided time series. On the other hand, owing in part to the orbital geometry and the short separation between the satellites, very low degree spherical harmonic coefficients may be not well constrained [Chen et al., 2005, 2006], even if latest GRACE postprocessing standards have much improved the reliability of their determination [Lombard et al., 2007].

[6] A valid alternative can be the analysis of SLR tracking that provides, in a consistent framework, precise determination of the temporal variation in the low-degree spherical harmonic components of Earth gravity field over a long time frame [Cheng et al., 1997]; moreover, SLR results provide a mechanism for validation of the GRACE temporal variability [Case et al., 2004]. In the last section, we provide a forward modeling of the expected perturbations to zonal variation rates in the years following the Sumatra earthquake for various mean asthenosphere viscosity values.

2. Data and Analysis

2.1. GRACE Data Processing

[7] We used the GRACE Level-2 Earth gravity field RL03 [Flechtner, 2005] and RL04 [Flechtner, 2007b] product releases provided by GeoForschungsZentrum (GFZ), one of the two central GRACE SDS centers. For each product release, data consists of 44 mean monthly estimates of fully normalized spherical harmonic coefficients (up to degree 120) of the geopotential for the period February 2003 to December 2006. We decided not to use the estimate relative to December 2004 to avoid anomalies connected with intersatellite range-rate change caused by the 2004 Sumatra tsunami [Bao et al., 2005; Chen et al., 2007]. The considered time window represents the overlapping period between RL03 and RL04 releases and therefore allowed us to compare in a consistent way the two GRACE solutions. Details on Level-2 data product releases history and on their modification can be found in the work of Flechtner [2007a, 2007b]. The monthly coefficients give information about changes caused by nonatmospheric and nonoceanic mass changes, i.e., mainly continental water storage changes, as well as other unmodeled geophysical effects, since all known contributions have been filtered out by the level-2 GRACE data processing acting on Level-1B raw telemetry data and other ancillary products [Bettadpur, 2007a, 2007b].

[8] As a first step, using the static field geopotential coefficients estimated from GSM products, we extracted the fully normalized \( C_{lm} \) and \( S_{lm} \) Stokes coefficients with degree \( l = 2, 3, 4, 5, 6 \) with their associated calibrated standard deviations. We decided to use harmonic degrees up to \( l = 6 \) in order to avoid the correlation noise found on higher spherical harmonics [Chen et al., 2007]. For harmonic degrees \( l = 2, 3, 4 \) the \( C_{20} \) coefficients provided by GRACE are corrected for a secular linear trend (\( \bar{C}_{20} \)) [Flechtner, 2007a, 2007b], so that to restore the full signal the subtracted trend has to be readded as follows (F. Flechtner, personal communication, 2007):

\[
\bar{C}_{20} = C_{20}^{corr} + \bar{C}_{20}(t - t_{ref})
\]

where the reference epoch \( t_{ref} \) is 1 January 1997 for release RL03 and 1 January 2000 for release RL04 [Flechtner, 2007b]. The rate \( \bar{C}_{20} \) is provided with the GRACE monthly solutions.

[9] Considering the relations [Hofmann-Wellenhof and Moritz, 2006]

\[
\bar{C}_{20} = \frac{1}{\sqrt{2l + 1}} C_{lm}; \quad J_l = -C_{l0}
\]

where \( C_{lm} \) are the unnormalized coefficients, we evaluated the zonal coefficients of the Earth gravity field as

\[
J_l = -\sqrt{2l + 1} \bar{C}_{20}
\]

In Figures 1 to 3 the time series of \( J_0, J_{lm}, \) and \( S_{lm} \), respectively, evaluated from RL03 as described above for the period February 2003 to December 2006, are represented by filled circles. The error bars represent the associated calibrated standard deviations provided by GRACE. The time series evaluated from RL04 with the same conditions are shown in Figures 4 to 6.

[10] Since the time series shown in Figures 1–6 above clearly contain a seasonal (annual and semiannual) signal due to large scale hydrological and ocean circulation cycles, we applied a filtering procedure to remove these periodic signals. As suggested by Moore et al. [2005], we fitted each harmonic coefficient with a linear term accounting for the secular trend plus sinusoidal terms at the annual and semiannual frequencies. In Figures 1 to 6 the solid gray curves represent the resulting fit. We then removed from each time history the annual and semiannual terms with amplitude and phase obtained from the fitting procedure described above.

2.2. Statistical Interpretation

[11] The expected signature of a giant earthquake on time histories of the gravity field harmonic coefficients is a coseismic jump followed by its postseismic relaxation. While the jump can be easily shadowed by occasional perturbations and by uncertainties [Gross and Chao, 2006], the postseismic relaxation could, in principle, be detectable as a perturbation to the secular trend of each coefficient. Therefore, our following step was to search for a signature of the seismic event on the time evolution of the coefficients; to this aim, we employed a statistical procedure in order to assess the existence of a given time \( t_0 \) (that we do not want to impose a priori) for which a statistically significant deviation of the time series from its secular trend is present. The basic idea is to check if a fit of the time series represented by two straight lines, which would model a trend variation at a given time, is significantly better than the one represented by a single straight line. A simple scheme of our procedure for a generic time series \( c(t) \) is
Figure 1. Monthly estimates of zonal coefficients $J_l$ ($l = 2, 3, 4, 5, 6$) with respect to February 2003 for the period February 2003 to December 2006 from RL03 products. Error bars represent the associated calibrated standard deviations from GRACE. The solid gray curve represents the annual plus semiannual plus linear fit. The vertical grey dash-dotted line represents $t = 26.12.2004$. 
Figure 2. Monthly estimates of $C_{lm}$ coefficients ($l = 2, 3, 4, 5, 6$) with respect to February 2003 for the period February 2003 to December 2006 from RL03 products. Error bars represent the associated calibrated standard deviations from GRACE. The solid gray curve represents the annual plus semiannual plus linear fit. The vertical grey dash-dotted line represents $t = 26.12.2004$. 
described here; it can be equivalently applied to $J_J$, $C_{lm}$ and $S_{lm}$ data sets.

1. We fit $c(t)$ with a linear function, $y_1(t) = a + bt$, through the linear least squares method; the standard deviations are assumed to be constant and unknown. Then we evaluate the associated chi-square, with $N/C0^2 = 44/C0^2$ degrees of freedom, where $m = 2$ is the number of fit parameters and $N = 44$ is the number of data points.

2. We fit the same data $c(t)$ with a set of bilinear functions defined as

\[
y_2(t, t_0) = \begin{cases} 
    a_1 + b_1 t, & t < t_0 \\
    a_2 + b_2 t, & t > t_0 
\end{cases}
\]

Figure 3. Monthly estimates of $S_{lm}$ coefficients ($l = 2, 3, 4, 5, 6$) with respect to February 2003 for the period February 2003 to December 2006 from RL03 products. Error bars represent the associated calibrated standard deviations from GRACE. The solid gray curve represents the annual plus semiannual plus linear fit. The vertical grey dash-dotted line represents $t = 26.12.2004$. 
Figure 4. Monthly estimates of zonal coefficients $J_l$ ($l = 2, 3, 4, 5, 6$) with respect to February 2003 for the period February 2003 to December 2006 from RL04 products. Error bars represent the associated calibrated standard deviations from GRACE. The solid gray curve represents the annual plus semiannual plus linear fit. The vertical grey dash-dotted line represents $t = 26.12.2004$. 
Figure 5. Monthly estimates of $C_{lm}$ coefficients ($l = 2, 3, 4, 5, 6$) with respect to February 2003 for the period February 2003 to December 2006 from RL04 products. Error bars represent the associated calibrated standard deviations from GRACE. The solid gray curve represents the annual plus semianual plus linear fit. The vertical grey dash-dotted line represents $t = 26.12.2004$. 
Figure 6. Monthly estimates of $S_{lm}$ coefficients ($l = 2, 3, 4, 5, 6$) with respect to February 2003 for the period February 2003 to December 2006 from RL04 products. Error bars represent the associated calibrated standard deviations from GRACE. The solid gray curve represents the annual plus semiannual plus linear fit. The vertical grey dash-dotted line represents $t = 26.12.2004$. 
For each $t_0$ varying monthly in the range from April 2003 to August 2006, we estimate the parameters $a_1, a_2, b_1,$ and $b_2$ with the least squares method and compute the resulting chi-square with $N-m = 44-4$ degrees of freedom. Also in this case we assume all standard deviations to be constant and unknown.

[14] For each time $t_0$, we want to assess the statistical significance of the fit improvement obtained with the bilinear function $y_2(t, t_0)$ with respect to the simple linear function $y_1(t)$. To this aim, we evaluate the chi-square values $\chi_1^2$ and $\chi_2^2(t_0)$, obtained with the $y_1$ and $y_2$ functions, respectively, and define the statistical variable

$$F_c(t_0) = \frac{\left[\chi_1^2(m) - \chi_2^2(t_0, m+2)\right]/2}{\chi_2^2(t_0, m+2)/(N-m-2)} = \frac{\Delta \chi_2^2(t_0)}{\chi_2^2(t_0)}$$

(4)

[15] Owing to the additive nature of functions that obey the $\chi^2$ statistics, this variable follows the Fisher $F$ distribution with $\nu_1 = 2$ and $\nu_2 = N - m - 2$ [Bevington and Robinson, 2003]. At each $t_0$, the value of $F_c(t_0)$ is a measure

Figure 7. $F_c(t)$ values for the $J_l$ time series ($l=2, 3, 4, 5, 6$) evaluated from GRACE RL03 products for the period April 2003 to August 2006, both (a) in case of standard deviations assumed unknown and constant and (b) in case of standard deviations estimated from GRACE. Horizontal lines show the confidence levels of 90% and 99%. The vertical grey dash-dotted line represents $t = 26.12.2004$. 
of how much the bilinear function \( y_2(t, t_0) \) improves the fitting of data with respect to the simple linear function \( y_1(t) \). Small values of \( F_c \) denote that the fit improvement is likely to be due just to a better adaptation to statistical errors, while large values of \( F_c \) indicate that the additional terms in fitting function \( y_2 \) are taken into account previously unmodeled data signals. It is to note that if the standard deviations of data points are assumed to be all equal, then their unknown values cancel out in the ratio of chi-squares in (4) and the chi-square can be safely replaced with the sum of squared residuals, i.e., \( \chi^2 = \sum (c(t) - y_{1,2}(t))^2 \); in this way there is no need to estimate the (unknown) standard deviations, which would introduce a bias in (4) vanishing its statistical significance.

[16] To assess quantitatively the improvement of the fit quality by adding the additional terms present in \( y_2 \), it is usual to estimate the integral probability \( P_F \) of observing by chance a value of \( F \) that is larger than \( F_N \), with \( \nu_1 \) and \( \nu_2 \) degrees of freedom, i.e.,

\[
P_F(F; \nu_1, \nu_2) = \int_0^\infty p_F(f; \nu_1, \nu_2) df
\]

and then the hypothesis of the presence of two distinct trends in the data is said to be accepted at \( (1 - P_F) \) confidence level.

[17] The same procedure was subsequently applied relaxing the assumption of equal unknown uncertainties and using the standard deviations from GRACE data. We remark that geopotential coefficient error estimates provided by GFZ, as for the estimates by the other SDS center, could differ depending on how the analysis was done [Bettadpur, 2007a]; besides, as observed by Tapley et al. [2004a], these estimates vary monthly in magnitude because of “[...] a combination of factors as ground-track coverage, temporal coverage (i.e., missing days), mismodeled short-period variability, and spacecraft events.” For these reasons, keeping the only assumption of equal unknown uncertainties in the linear least squares fitting might be the most suitable choice, since standard deviations from GRACE data, being probably underestimated, could not be representative of variable uncertainties in a proper statistical sense (F. Flechtner, personal communication, 2007).

[18] Figure 7 shows, for the period April 2003 to August 2006, the value of \( F_c(t_0) \), computed as described above, for the \( J_l \) time series (\( l = 2, 3, 4, 5, 6 \)) from RL03 both in case of constant unknown standard deviations (Figure 7a) and in case of GRACE standard deviation estimates (Figure 7b). The horizontal lines indicate \( F \) values corresponding to 90% and 99% confidence level as defined in (5). The \( F_c(t_0) \), computed for the \( J_l \) time series from RL04 with the same procedure described above, is shown in Figure 8.

[19] From Figures 7 and 8, for almost all of the considered harmonic degrees, we get large values of \( F_c \) at the time of the earthquake. If we look at the temporal dependence of \( F_c(t) \), we see that for some of the harmonic degrees we have local peaks shortly before or shortly after the occurrence of the Sumatra event (vertical grey dash-dot line in Figures 7 and 8). These peaks are more pronounced when standard deviations are estimated from data with respect to the analysis performed with GRACE estimates of standard deviations. From this evidence we can argue that there is a statistically significant improvement of the fit when using a bilinear function with a breakpoint \( t_0 \) at the occurrence of the Sumatra event with respect to a fit with a single linear function, but at the same time with current data we are not able to precisely resolve the breakpoint time without imposing it a priori. When comparing the results obtained with the two GRACE solutions (RL03 and RL04), we can see that while the significance of the \( F_c(t) \) is more marked for results from RL03, the trends of \( F_c(t) \) from the two release products are qualitatively similar, both in case of constant unknown standard deviations and in case of standard deviations from GRACE. This is an indirect confirmation of the stability of our statistical analysis, since we are obtaining equivalent results with different analysis procedures.

[20] In order to take into account simultaneously all the harmonic terms, we computed a global \( F_c(t_0) \), by applying (5) to cumulative chi-squares obtained with all considered time series. The results are shown in Figure 9 for \( J_l \), \( C_{lm} \) and \( S_{lm} \) coefficients. We see again that with the considered analysis it is not possible to resolve a preferred change point \( t_0 \), since the peak region turns out to be very broad even if it is centered around the Sumatra earthquake occurrence time. A similar conclusion can be drawn from the analysis of cumulative residuals of the fits (Figure 10), which show a broad minimum centered around the earthquake occurrence time. From Figures 9 and 10 we can also see that there is no substantial difference between RL03 and RL04 and between the analysis performed with GRACE error estimates and with constant unknown errors, suggesting that the observed features are intrinsic to data and not to the analysis details.

[21] In Table 1 we report the values of \( (1 - P_F) \), being \( P_F \) the integral probability as described in (5) with \( F = F_c(t_0) \) and \( t_0 \) corresponding to the earthquake time, evaluated for the \( J_l \) time series (\( l = 2, 3, 4, 5, 6 \)) from RL03 and RL04 release products, both in case of constant unknown standard deviations and in case of standard deviations estimated from GRACE. It is important to emphasize the absolute values of \( (1 - P_F) \) listed in Table 1: they represent the confidence level at which we accept the hypothesis of a deviation of the \( J_l \) time series from its secular trend, just at Sumatra event time. We see from the confidence levels that the trend variation is statistically significant in all cases except for \( l = 5 \).

[22] From the statistical analysis described above, we can therefore summarize that if we introduce a perturbation in \( J_l \) temporal drifts caused by postseismic relaxation of the Sumatra earthquake, we get a statistically significant improvement of the fit, but we cannot avoid the a priori imposition of the drift change point (coincident with earthquake occurrence time) because it is not possible to resolve it from data.

2.3. Trend Variation

[23] To compute the secular trend perturbation in correspondence of the Sumatra earthquake, we fitted the GRACE time series from RL03 and RL04 product releases for the period February 2003 to December 2006 with a set of bilinear functions, as defined in (3), setting the temporal linear drift change point \( t_0 \) equal to the one correspondent to November 2004 estimates of the coefficients, just before the earthquake. For the reasons previously discussed, also in
this case we do not use the estimate relative to December 2004. The slope variation, $\Delta s_{\text{grace}}$, was evaluated as the difference between the slope value of the linear fit after $t_0$ and the slope value of the linear fit before $t_0$, i.e., $b_2 - b_1$ in equation (3). In Figures 11 and 12 we show $\Delta s_{\text{grace}}$ evaluated from $J_l$ time series ($l = 2, 3, 4, 5, 6$) from RL03 and RL04, respectively. Also in this part of the analysis, for the estimation of the statistical error associated with $\Delta s_{\text{grace}}$, $\sigma_{\Delta s_{\text{grace}}} = \sqrt{\sigma_{b_1}^2 + \sigma_{b_2}^2}$, we discriminated the case of constant unknown standard deviations (Figures 11a and 12a) from the case of GRACE estimates of standard deviations (Figures 11b and 12b). The corresponding numerical values are listed in Table 2, where we report also the combined estimate of $\Delta s_{\text{grace}}$ obtained as a weighted average of the results obtained with RL03 and RL04. We can observe, within the associated errors, a negative slope variation on all harmonic degrees considered here both for RL03 and for RL04; besides, for both releases, the dependence of $\Delta s_{\text{grace}}$.
on the harmonic degree is qualitatively similar between the cases of unknown errors and of GRACE error estimates; this is a further confirmation of the stability of our statistical analysis. From Table 2 we can see that the slope perturbations obtained as weighted averages of the RL03 and RL04 data sets are not null at 1-σ confidence level for all the harmonic degrees except \( l = 5 \), both when using GRACE estimates of standard deviations and when assuming unknown standard deviations. The harmonic degree \( l = 5 \), for which we get a slope perturbation statistically compatible with zero, was the only one for which we obtained low values of \( F \) in the statistical test we presented in the previous section. The estimates of \( \Delta s_{\text{GRACE}} \) obtained with RL04 have amplitude generally smaller or equal with respect to the corresponding values obtained with RL03, while the uncertainties are comparable; as a result, the RL04

Figure 9. Global \( F_c(t) \) values obtained simultaneously with all the (top) \( J_l \), (middle) \( C_{lm} \), and (bottom) \( S_{lm} \) time series, evaluated from GRACE products for the period April 2003 to August 2006. Horizontal lines show the confidence levels of 90% and 99%. The vertical grey dash-dotted line represents \( t = 26.12.2004 \).
Figure 10. Cumulative residuals obtained with the (top) $J_h$, (middle) $C_{lm}$, and (bottom) $S_{lm}$ time series, evaluated from GRACE products for the period April 2003 to August 2006. The vertical grey dash-dotted line represents $t = 26.12.2004$. Residuals are normalized to the value obtained with $t = 26.12.2004$. 
results turn out to be slightly less significant. When comparing the results obtained with GRACE estimates of errors with those obtained with unknown standard deviations, we note that the two results are fairly different but still remain compatible within the associated errors. Incidentally, when using GRACE estimates of standard deviations we obtain smaller values of $\sigma_{D_{\text{grace}}}$; this can be interpreted as an effect of a coefficient error underestimation [Wahr et al., 2006]. As a final note, we verified that we got substantially the same results, if we consider a temporal drift change point $t_0$ correspondent to January 2005 estimates of the $C_{lm}$ coefficients, just after the earthquake. 

Table 1. Values of $(1 - P_{\text{F}_c})$ Corresponding to $F_{\text{c}}(t)$ Evaluated at the Earthquake Occurrence Time for the $J_l$ Time Series Evaluated From RL03 and RL04 Products$^a$

<table>
<thead>
<tr>
<th></th>
<th>RL03</th>
<th>RL04</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_2$</td>
<td>0.99</td>
<td>0.96</td>
</tr>
<tr>
<td>$J_3$</td>
<td>0.89</td>
<td>0.84</td>
</tr>
<tr>
<td>$J_4$</td>
<td>1.00</td>
<td>0.98</td>
</tr>
<tr>
<td>$J_5$</td>
<td>0.77</td>
<td>0.98</td>
</tr>
<tr>
<td>$J_6$</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

$^a$Both cases of unknown constant standard deviations ($\sigma_u$) and of GRACE estimates of standard deviations ($\sigma_{G\text{grace}}$) are shown.

Figure 11. $\Delta s_{\text{grace}}$ with its associated error for $J_l$ time series evaluated from GRACE RL03, both (a) in case of standard deviations assumed unknown and constant and (b) in case of standard deviations estimated from GRACE.

Figure 12. $\Delta s_{\text{grace}}$ with its associated error for $J_l$ time series evaluated from GRACE RL04, both (a) in case of standard deviations assumed unknown and constant and (b) in case of standard deviations estimated from GRACE.

[24] In Figures 13 and 14 we show the secular trend variation $\Delta s_{\text{grace}}$ estimated from GRACE data for nonzonal $C_{lm}$ and $S_{lm}$ time series, respectively. In this case, many of the trend variations are statistically compatible with zero and there are no general features as we found for the $J_l$ trend perturbations. While results obtained using the GRACE error estimates are generally close to those obtained assuming constant unknown errors, there is no general agreement between results obtained with RL03 and RL04, with the
Table 2. $J_l(t)$ Secular Trend Perturbation With its Associated Error Evaluated Assuming a Drift Change Point at November 2004\textsuperscript{a}

<table>
<thead>
<tr>
<th></th>
<th>RL03</th>
<th></th>
<th>RL04</th>
<th></th>
<th>RL03 + RL04</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_2$</td>
<td>$-7.02 \pm 1.56$</td>
<td>$-6.11 \pm 3.86$</td>
<td>$-3.79 \pm 1.29$</td>
<td>$-1.51 \pm 3.64$</td>
<td>$-3.68 \pm 2.65$</td>
<td>$-5.10 \pm 0.99$</td>
</tr>
<tr>
<td>$J_3$</td>
<td>$-1.78 \pm 0.46$</td>
<td>$-3.82 \pm 1.97$</td>
<td>$-1.83 \pm 0.38$</td>
<td>$-3.25 \pm 1.54$</td>
<td>$-3.47 \pm 1.21$</td>
<td>$-1.81 \pm 0.29$</td>
</tr>
<tr>
<td>$J_4$</td>
<td>$-4.71 \pm 0.38$</td>
<td>$-4.31 \pm 0.81$</td>
<td>$-1.47 \pm 0.32$</td>
<td>$-1.07 \pm 0.74$</td>
<td>$-2.54 \pm 0.55$</td>
<td>$-2.81 \pm 0.24$</td>
</tr>
<tr>
<td>$J_5$</td>
<td>$-0.35 \pm 0.24$</td>
<td>$+0.73 \pm 0.83$</td>
<td>$-0.98 \pm 0.20$</td>
<td>$-0.16 \pm 0.77$</td>
<td>$+0.25 \pm 0.56$</td>
<td>$-0.72 \pm 0.15$</td>
</tr>
<tr>
<td>$J_6$</td>
<td>$-2.04 \pm 0.21$</td>
<td>$-2.31 \pm 0.57$</td>
<td>$-2.18 \pm 0.17$</td>
<td>$-2.16 \pm 0.36$</td>
<td>$-2.20 \pm 0.30$</td>
<td>$-2.12 \pm 0.13$</td>
</tr>
</tbody>
</table>

\textsuperscript{a}Values obtained with releases RL03 and RL04 and a weighted average of RL03 + RL04 are shown, both in case of unknown constant standard deviations ($\sigma_u$) and of GRACE estimates of standard deviations ($\sigma_G$). Numerical values are in units of $10^{-13}$ a$^{-1}$.

Figure 13. $\Delta_{\text{GRACE}}$ with its associated error for $C_{lm}$ time series evaluated from (top) GRACE RL03 and (bottom) RL04, both in case of standard deviations assumed unknown and constant ("unknown $\sigma$") and in case of standard deviations estimated from GRACE ("known $\sigma$").
exception of $l=6$ coefficients. In this respect, we can conclude that the evidence for a trend perturbation in $C_{lm}$ and $S_{lm}$ nonzonal coefficients is much weaker; this can be a consequence of the fact that the effect of the zonal harmonics on satellite orbits is much greater than that of the other harmonics [Hofmann-Wellenhof and Moritz, 2006] and therefore the $J_l$ terms are the most precisely measured by satellite missions.

3. Postseismic Relaxation Modeling

[25] The postseismic perturbation to the gravitational potential due to solid mass displacements induced by the Sumatra earthquake, in terms of relaxation of the ductile asthenosphere, is expected to give a continuous temporal variation of harmonic coefficients. On temporal scales much shorter than the characteristic relaxation times of the system, the coefficient variation will be well approximated by a linear law [Piersanti et al., 1995], whose slope will represent a perturbation to the secular trend. In the following analysis we modeled the temporal evolution of the harmonic coefficients for various asthenosphere viscosity values; in this way, we want to ascertain if the asthenospheric viscosity is able to leave a detectable signature on measured time histories, and if the slope variations obtained in section 2.3 can be used to infer indications on its mean value.

[26] We computed the time-dependent evolution of low-degree zonal coefficients of the Earth gravity field associated with the 2004 Sumatra earthquake adopting the theoretical model originally proposed by Piersanti et al.
Figure 15. Modeled temporal evolution of $J_l$ ($l = 2, 3, 4, 5, 6$) with respect to January 2005 for the period January 2005 to December 2007. Different marker symbols represent different values of asthenosphere viscosity.
[1995] and later refined by Soldati et al. [1998] and by Boschi et al. [2000]. It is a semianalytical, spherical model which assumes an incompressible, layered, self-gravitating Earth with Maxwell linear viscoelastic rheology; it allows to compute physical observables such as deformation, geopotential, and stress tensor. In its current formulation, the model neglects any contribution from water load effects.

[27] We employed a four-layer stratification model with an 80 km thick elastic lithosphere, a 200 km asthenosphere with variable mean viscosity values, a uniform viscoelastic mantle with a constant viscosity value of $10^{21}$ Pa s and a fluid inviscid core. All the other mechanical parameters have been obtained by means of a weighted volume average of the corresponding parameters of the Preliminary Reference Earth Model (PREM) [Dziewonski and Anderson,
1981]. It is to note that while the PREM model is compressible, the analytical formulation of our model is based on an incompressible rheology; that is to say, we adopt a modified version of PREM with $\lambda \rightarrow \infty$. This approximation certainly affects our results, as discussed in detail by Nostro et al. [1999], but it allows us to take into account simultaneously without any kind of approximation viscoelasticity, sphericity and self-gravitation, the role of the latter being of particularly relevance in our analysis since we focus on the modeling of gravitational terms.

[28] The 2004 Sumatra seismic source has been modeled using the multiple CMT solution provided by Tsai et al. [2005], which was obtained by fitting with the CMT method [Dziewonski et al., 1981] the long-period seismograms from the IRIS Global Seismographic Network and accounts for a cumulative energy release corresponding to $M_w = 9.3$.

[29] We evaluated the temporal evolution of the perturbation to $C_{lm}$ and $S_{lm}$ due to the postseismic relaxation of the Sumatra earthquake for asthenospheric viscosity values $\eta_1 = 10^{16}$, $\eta_2 = 10^{17}$, $\eta_3 = 10^{18}$ and $\eta_4 = 10^{21}$ Pa s, in a time window beginning from January 2005 up to December 2007, with equal time intervals of about 1 month, in order to simulate monthly intervals of GRACE time histories. The chosen asthenospheric viscosity values are representative of different relaxation regimes, whose inference is widely discussed in mantle rheology literature [Pollitz et al., 1998; Piersanti, 1999; Kaufmann and Lambeck, 2002]. In particular, the values of $\eta_1$ and $\eta_2$ have been chosen because they can approximate, on the considered timescale, the effect of a Burgers transient rheology [Yuen and Pelletier, 1982]; indeed, for small timescales, the relaxation of a Burgers body is dominated by the transient effect of the low-viscosity Kelvin-Voigt element.

[30] In Figure 15 we show the expected postseismic perturbation to $J_l(t)$ for $l = 2, 3, 4, 5, 6$ with the considered viscosity values. First of all, we note that for $\eta_4 = 10^{21}$ Pa s the zonal term $J_6(t)$ is almost constant on the considered scale; instead, for the other viscosities, $J_l(t)$ show a monotonic trend, increasing for $\eta_1$ and decreasing for $\eta_2$ and $\eta_3$. Therefore, we may expect that for large asthenospheric viscosities the perturbation to $J_l(t)$ secular trend will result too small to be discriminated.

[31] For all the considered harmonic coefficients, we computed the modeled perturbation to the secular trend, $\Delta s_{\text{model}}$, as the angular coefficient of a straight line fitting the curve. We used a linear regression since it is a fair approximation, given the quasi-linear trend of the postseismic relaxation immediately after an earthquake [Piersanti et al., 1995]. Indeed, we were able to verify the stability of the trend estimate by increasing the time window up to December 2010 and observing a substantial invariance of the slope.

[32] We compare the values obtained from GRACE RL03 and RL04 data and the modeled predictions, through evaluation of the weighted residuals computed using the associated statistical errors, i.e.,

$$D = \frac{\Delta s_{\text{grace}} - \Delta s_{\text{model}}}{\sigma_{\Delta s_{\text{grace}}}}$$

where $\Delta s_{\text{grace}}$ is the slope variation of the bilinear fitting functions in GRACE time series, as defined previously in section 2.3, $\Delta s_{\text{model}}$ is the modeled slope perturbation induced by Sumatra earthquake, i.e. the slope of the straight line fitting the modeled coefficient time history, and $\sigma_{\Delta s_{\text{grace}}}$ is the associated standard deviation, also defined in section 2.3.

[33] In Figures 16 and 17 we show the values of $D$ for the $J_l$ coefficients from RL03 and RL04, respectively, as a function of the harmonic degree, for all the considered asthenospheric viscosity values ($\eta_1 = 10^{16}$, $\eta_2 = 10^{17}$, $\eta_3 = 10^{18}$ and $\eta_4 = 10^{21}$ Pa s), both in case of constant unknown standard deviations (Figures 16a and 17a) and in case of standard deviations from GRACE (Figures 16b and 17b). We have to observe that for each product release, when using standard deviations from GRACE data, the residual $D$ is about double with respect to the case of unknown standard deviations. This is a consequence of the small values of $\sigma_{\Delta s_{\text{grace}}}$ obtained if standard deviations from GRACE are taken into account, as we discussed in section 2.3; anyway, we do not have elements to further speculate about the nature of this behavior, which is related to the details of GRACE data processing. Moreover, the relative proportions between residuals at different harmonic degrees are almost unchanged between the two cases.

[34] If we compare the results obtained with the two product releases (RL03 and RL04), we see that the lowest residuals are obtained with RL04, where in the case of unknown standard deviations (Figure 17a), we get residuals $D \leq 2.5$ for all viscosity values and harmonic degrees, except for $l = 6$, where we get $D$ ranging from 3 to 6.5 depending on the asthenospheric viscosity.

[35] For what concerns the effect of the mean asthenospheric viscosity, we see that the lowest weighted residuals are obtained for a viscosity value equal to $\eta_2 = 10^{17}$ Pa s, followed by $\eta_3 = 10^{18}$ Pa s, except for $l = 5$, both for RL03 and RL04. Incidentally, we remark that the harmonic term $l = 5$ is the only one for which we obtained a low statistical significance of the trend variation (see Table 1). For the best fitting model, which has mean viscosity $\eta_2 = 10^{17}$ Pa s, we obtain an agreement within $1.5\sigma$ with respect to data for low harmonic degrees ($l \leq 4$). Anyway, it must be noted that there is not a marked difference between the residuals obtained with the other viscosity model; in this respect, we can conclude that the observed slope variation is in reasonable agreement with our model, but we cannot use it to discriminate between various mean asthenospheric viscosities because there is no sufficient sensitivity. Nevertheless, we get the best agreement with data when using a mean asthenospheric viscosity on $10^{17}$ Pa s, that can be addressed in terms of a short-timescale equivalent behavior of a Burgers transient relaxation process.

[36] Furthermore, we note that our best fitting viscosity range is in good agreement with the average Maxwell viscosity of the oceanic asthenosphere inferred by Pollitz et al. [1998] and with the short-term viscosity used by Pollitz et al. [2006] in the postseismic modeling of the Sumatra earthquake.

[37] In Figures 18 to 21, we show the residuals $D$ obtained for nonzonal $C_{lm}$ and $S_{lm}$ time histories as a function of the mean asthenospheric viscosity. Residuals are shown for data releases RL03 and RL04, obtained both using GRACE error estimates and assuming unknown
constant errors. From these results we see that there is no appreciable difference between residuals obtained with different viscosity models. As observed in the discussion above, the residuals obtained when assuming constant unknown standard deviations are generally smaller than the ones obtained using GRACE error estimates, with some exceptions for the highest harmonic degrees. For the lowest degrees ($l = 2, 3$) we obtain the smallest residuals with RL03 and constant unknown errors (Figure 19); for the higher degrees we find coefficients with large residuals under all the four analysis conditions, even if in the majority of the cases the modeled slope perturbation falls within $\sim 2\sigma$ from the observed one. Summarizing these results, we can conclude that for nonzonal terms we are not able to find a general agreement of modeled slope perturbations with observed data; as we speculated above, one of the possible

Figure 17. Data-model weighted residuals for $J_l$ time series, both (a) in case of standard deviations assumed unknown and constant and (b) in case of standard deviations estimated from GRACE RL04 products. Different marker symbols represent different values of asthenosphere viscosity.
4. Signature on SLR Observations

[38] Because of the sensitivity yielded by satellite laser ranging (SLR) technique on measurements of the temporal changes in the Earth’s gravity field and in particular of the temporal variability in the lower-degree harmonics [Cheng et al., 1997; Nerem et al., 2000], we suggest a possible detection of the postseismic change in gravitational field harmonics through SLR data observation. In fact, the long temporal baseline of the SLR tracking data is able to provide a measurement of the secular variation in the low-degree zonal harmonics with a precision that cannot be reached by satellite missions as GRACE.

[39] In a previous paper, Cannelli et al. [2007] modeled the perturbation to $J_2$ variation rate induced by postseismic relaxation following the Sumatra earthquake. As a complement to the analysis described in the previous section, we extend to harmonic degrees up to 6 the preliminary analysis made in the work of Cannelli et al. [2007].

[40] We computed the temporal evolution of the $J_l$ variation rate in a 10-year time window beginning from the Sumatra event, for asthenospheric viscosity values ranging from $10^{15}$ to $10^{22}$ Pa s. The time-dependent $J_l$ variation rate, $J_l$, was evaluated as the difference between $J_l$ coefficients computed for subsequent times, spaced at equal intervals of about 3 months, i.e., $J_l(t_i) = (J_l(t_i) - J_l(t_{i-1}))/t_i - t_{i-1}$. In

![Figure 18. Data-model weighted residuals for $C_{lm}$ and $S_{lm}$ time series, obtained with GRACE RL03 products assuming unknown and constant standard deviations. See also legend of Figure 16.](image)
Figure 22, we show the temporal evolution of \( J_l \) over a period of 10 years for \( l = 2, 3, 4, 5, 6 \) and for asthenospheric viscosities \( \eta_1 = 10^{16}, \eta_2 = 10^{17}, \eta_3 = 10^{18} \) and \( \eta_4 = 10^{19} \) Pa s. As can be seen, low asthenospheric viscosities yield very large \( J_l \) in the first years after the event; for subsequent times the difference between results obtained with the various viscosities is less marked but might be anyway detectable. From this point of view, these results can in principle be used to put a lower limit for the asthenospheric viscosity on the basis of geodetic measurements of \( J_l \). In fact, if a detection threshold for deviations of \( J_l \) from its secular drift is assumed, we can assess for which viscosity ranges a detectable signal from available SLR data is expected in the next years.

Figure 19. Data-model weighted residuals for \( C_{lm} \) and \( S_{lm} \) time series, obtained with GRACE RL03 products using GRACE standard deviations estimates. See also legend of Figure 16.

\[41\] As can be seen, low asthenospheric viscosities yield very large \( J_l \) in the first years after the event; for subsequent times the difference between results obtained with the various viscosities is less marked but might be anyway detectable. From this point of view, these results can in principle be used to put a lower limit for the asthenospheric viscosity on the basis of geodetic measurements of \( J_l \). In fact, if a detection threshold for deviations of \( J_l \) from its secular drift is assumed, we can assess for which viscosity ranges a detectable signal from available SLR data is expected in the next years.

\[42\] As a reasonable value for the detectability threshold of perturbations to \( J_2 \) we chose the formal error associated to its secular value, which is represented in Figure 22 as a shaded area. For \( l > 2 \), the modeled signals are always included within the experimental error associated to secular rates given by Cheng et al. [1997]. For asthenospheric viscosities lower than \( 10^{18} \) Pa s, the effect of the Sumatra event in \( J_2 \) would remain detectable for several years, while the signals associated with greater viscosities lie marginally or below to the detectability threshold. Also for higher order zonals we expect a remarkably large perturbation to zonal rates for viscosities below \( 10^{18} \) Pa s even if, as we stated above, this signal may still be not detectable due to large experimental errors associated with \( l > 2 \) zonals [Cheng et
Anyway, we expect that a more precise determination of these zonal rates may be possible with a more complete temporal coverage.

Table 3 lists the expected values of the mean annual variation rate \( J_l \) for \( l = 2, 3, 4, 5, 6 \) during 2007 and 2008 for asthenosphere viscosities ranging from \( 10^{15} \) to \( 10^{22} \) Pa s. It can be seen as for viscosities lower than \( 10^{18} \) Pa s we expect that SLR data will be affected by a strong signature with respect to the secular linear drift value estimated for each \( J_l \); instead for viscosities greater than \( 10^{19} \) Pa s the signal seems to be marginal or even below the detectability threshold and therefore not so evidently detectable.

This indication, that we remark is coming from a forward modeling, could allow to perform an indirect estimate of asthenospheric viscosity or alternatively, if no signal is observed, to put a lower limit on its mean value.

5. Conclusions

In this work we suggest the hypothesis that the effect of the internal mass redistribution due to the Sumatra earthquake on the Earth gravity field would leave a detectable signature on the low-degree gravity harmonics provided by GRACE.

We found a statistically significant deviation of the low-degree \( J_l \) time series from their secular trend, in terms of an improvement in the fit of GRACE time series when a bilinear function rather than a straight line is used; the
variation to zonal rates obtained with RL03 and RL04 products are different but still statistically compatible within their associated uncertainties. A similar perturbation is found in nonzonal coefficient time series, but its statistical significance turns out to be weaker.

When we assume the standard deviations of harmonic coefficients to be constant and unknown, we find a slope reduction on $J_l$ after the Sumatra event for all the considered degrees, except for $l = 5$. In particular, for $J_2$ we find the largest variation, which corresponds to about 2.2% of its secular rate. If using GRACE estimates of standard deviations, the $J_2$ slope variation is less pronounced (about 0.4% of its secular rate) but it is anyway negative. A negative variation occurs for all the other degrees except for $l = 3$.

The combined estimate of the slope variation obtained using both RL03 and RL04 data turns out to be negative and not null at 1-$\sigma$ confidence level for all harmonic degrees except $l = 5$. The statistical analysis confirmed that these variations are significant, but current data is not able to resolve the drift change point which has to be imposed at the earthquake occurrence time.

It is evident that lifetime of GRACE mission sets a definitive superior limit to the temporal window; the planned 5-year mission life has been extended by at least 2 more years [Massmann et al., 2007], so the availability of $J_l$ time histories on a longer time window will enable us to further reduce the uncertainties associated with the present analysis.

We modeled the postseismic evolution of the perturbation to $J_l$ coefficients induced by the Sumatra earthquake, as a function of asthenosphere viscosity, since the gravity field reflects the mass inhomogeneities in the Earth’s

Figure 21. Data-model weighted residuals for $C_{lm}$ and $S_{lm}$ time series, obtained with GRACE RL04 products using GRACE standard deviations estimates. See also legend of Figure 16.
interior and can be used to understand and improve models for the internal structure. The comparison between the modeled perturbation to the secular trend of $J_l$ time evolution and the trend variation of $J_l$ time series from GRACE observations in correspondence of Sumatra earthquake is not able to give a precise indication of a preferred value of asthenosphere viscosity, even if low asthenospheric viscosities ($\eta = 10^{17} \div 10^{18} \text{ Pa s}$) give the lowest residuals. In this respect, we may conclude that the observed trend perturbation following the Sumatra earthquake is consistent with our model of postseismic relaxation but we cannot use it as a discriminating tool between different asthenospheric viscosity models; the fact that the best fit is obtained with low viscosities may be interpreted as the effect of the transient relaxation of a biviscous asthenosphere with Burgers rheology, that on short timescale may be approximated with a low-viscosity Maxwell rheology. It is also to note that for asthenospheric viscosity values greater or equal than $10^{17} \text{ Pa s}$, our model predicts a concordant negative perturbation to $J_2(t)$ for all the zonal gravitational field harmonics considered here, in agreement with the sign of slope variation observed in GRACE time series.

[50] A further insight into the postseismic effects on gravitational parameters may come from satellite laser

**Figure 22.** Time evolution of $J_l$ variation rate ($dJ_l/dt$) for $l = 2, 3, 4, 5, 6$. Different line styles represent different values of asthenosphere viscosity. The shaded area in the top left panel corresponds to values of $J_2$ below the detectability threshold.
ranging observations, which provide longer time histories with respect to satellite missions and therefore can be used to extract the secular drift from the higher frequency variations in the low-deegree zonal terms with smaller uncertainties. As a complement to the analysis of GRACE data, we performed a forward modeling of the effect of the postseismic relaxation of the Sumatra event on $J_l$ time evolution as a function of mean asthenospheric viscosity. We found that values of $\eta$ lower than $10^{18}$ Pa s give rise, in the years following the event, to a remarkably large detectable deviation of $J_l$ from its secular drift, that for $l = 2$ is above the secular rate uncertainty level. These results suggest that from an ad hoc analysis of SLR tracking data it may be possible to associate a perturbation of $J_l$ after the Sumatra event to a mean value of asthenospheric viscosity or, if no such perturbation is observed, to put a lower limit on the value of $\eta$. This kind of analysis is not currently possible with publicly available data due to large uncertainties on $l = 2$ harmonics, but a more precise analysis may be feasible with the availability of longer $J_l$ time histories.

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